Three-dimensional micromagnetic finite element simulations including eddy currents

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We developed a micromagnetic eddy current method that allows arbitrary geometries, requires no mesh outside the ferromagnet, and uses a stable integration scheme. We simultaneously solve the Landau–Lifshitz–Gilbert equation and the quasistatic Maxwell equations using a hybrid finite element/boundary element method (FEM/BEM). The eddy current field is directly calculated from the space time behavior of the magnetization rate of change. The boundary conditions of the eddy current field at infinity are taken into account using a FEM/BEM scheme. The resulting system of differential algebraic equations is solved using a backward differentiation method. © 2005 American Institute of Physics. [DOI: 10.1063/1.1852211]

I. INTRODUCTION

Eddy current effects are usually not taken into account in standard magnetic calculations that are based on assumptions of quasistatic approximation, small material conductivity or that the contribution of eddy currents is included in the dimensionless damping constant of the Landau–Lifshitz equation.

The main motivation to develop a dynamic micromagnetic model that includes eddy currents is the importance of accurate calculations of time dependent micromagnetic processes for magnetic data storage (hard disk recording magnetic random access memory MRAM) at high data rates and the demand for ultrafast switching in magnetic nanostructures. Especially for magnetic recording heads with field rise times of less than 0.1 ns and a conductivity $\sigma$ of less than $0.6 \times 10^7 \, (\Omega \, m)^{-1}$, the assumptions above are no longer applicable.

Several eddy current methods have been presented in the past. These include the two-dimensional quasistatic model by Della Torre and Eicke and the one-dimensional dynamic calculations by Sandler and Bertram. A hybrid method for three-dimensional eddy current problems that is based on the solution of the differential equation for the current density and the magnetic field where the Bio-Savart law is used to calculate the field intensity of the magnetic field on the surface was introduced by Kalimov and co-workers. Serpico and co-workers developed a finite difference scheme that is applied to the analysis of eddy currents with the Landau–Lifshitz equation as a constitutive relation. We developed a three-dimensional finite element dynamic micromagnetic model including eddy currents. Our model simultaneously solves the Landau–Lifshitz–Gilbert (LLG) equation and the quasistatic Maxwell equation using a hybrid finite element/boundary element (FE/BE) method. A FE method with a linear basis function is used to discretize the conducting region $\Omega$. To calculate the magnetization vector on each node of our mesh an additional discretization was used to calculate the exact volume around each node. The boundary element method is used to map the boundary conditions of the magnetic field at infinity on equivalent boundary conditions on the surface of the conducting region $\Omega$, which reduces the necessity of a mesh to $\Omega$. The eddy current field is introduced as an additional part of the effective magnetic field in the LLG equation, and is directly calculated from the space time behavior of the magnetization rate of change.

II. MODEL

To formulate our eddy current model we consider a conducting region $\Omega$ with the conductivity $\sigma$ and the magnetic permeability $\mu$. Eddy currents are induced inside the region by applying an external magnetic field which changes the magnetization.

The dynamic magnetization process in the conducting region is described by the LLG equation of motion

$$ \frac{\partial \mathbf{m}}{\partial t} = -\gamma' \mathbf{m} \times \mathbf{H}_{\text{eff}} - \frac{\alpha' \gamma'}{m_s} \mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{\text{eff}}) $$

with

$$ \gamma' = \frac{\gamma}{1 + \alpha^2}, $$

where $\alpha$ is a dimensionless damping constant and $\gamma$ is the electron gyromagnetic ratio. The effective field $\mathbf{H}_{\text{eff}}$ is composed of the anisotropy field, the exchange contribution, the applied field, the magnetostatic field, and the eddy current field, which is introduced to take eddy current effects into account.
The applied field $H_a$ is a solenoidal field and is produced by an external current $j_a$. The magnetostatic field $H_M$ is irrotational as its source is the divergence of the magnetization $M$. The introduced eddy current field $H_{eddy}$ is a solenoidal field that is produced by eddy currents $j_{eddy}$.

Summarized conditional equations are as follows:

$$
\nabla \cdot H_a = 0, \quad (4)
$$

$$
\nabla \times H_a = j_a, \quad (5)
$$

$$
\nabla \cdot M = - \nabla \cdot \mathbf{M}, \quad (6)
$$

$$
\nabla \times M = 0, \quad (7)
$$

$$
\nabla \cdot H_{eddy} = 0, \quad (8)
$$

$$
\nabla \times H_{eddy} = j_{eddy}. \quad (9)
$$

To calculate the eddy current field we take the curl of Eq. (9) and use Faradays law to get

$$
\nabla^2 H_{eddy} = \frac{\sigma}{\partial t} B, \quad (10)
$$

Considering that $\mathbf{B} = \mu_0 (H + M)$ and that for soft magnetic materials the applied and magnetostatic fields are much smaller than the magnetization $M$ we get

$$
\nabla^2 H_{eddy} - \frac{\sigma \mu_0}{\partial t} \frac{\partial H_{eddy}}{\partial t} = \frac{\sigma \mu_0}{\partial t} \frac{\partial M}{\partial t}. \quad (11)
$$

To solve the coupled LLG equation and the quasistatic Maxwell equations we use the hybrid FE/BE method as described by Fredkin and Koehler. For the region $\Omega$ we use a finite element method with the Ritz-Galerkin weak formulation. For the region outside $\Omega$ the finite element method problem with boundary conditions at infinity are mapped onto an equivalent boundary element method problem with boundary conditions on the surface $\partial \Omega$ of the conducting region. Thus, no finite elements outside $\Omega$ are needed.

The Poisson equation for the eddy current field is

$$
\nabla^2 H_{eddy} = \left\{ \begin{array}{ll}
\sigma \mu_0 \frac{\partial H_{eddy}}{\partial t} + \sigma \mu_0 \frac{\partial M}{\partial t} & \Omega \\
0 & \mathbb{R}^3 \setminus \Omega
\end{array} \right. \quad (12)
$$

with the boundary condition at infinity $H_{eddy} = 0$. The boundary condition on $\partial \Omega$ is

$$
n \times H_{eddy}^{in} - n \times H_{eddy}^{out} = 0, \quad (13)
$$

where $n = n(r)$ is the unit vector at $r$ pointing outward with respect to the boundary of $\Omega$. Using the linearity of Eq. (12) the solution is split into two parts

$$
H_{eddy} = H_{eddy}^1 + H_{eddy}^2, \quad (14)
$$

where $H_{eddy}^1$ is a particulate solution that solves the inhomogeneous boundary problem

$$
\nabla^2 H_{eddy}^1 = \sigma \mu_0 \frac{\partial H_{eddy}^1}{\partial t} + \sigma \mu_0 \frac{\partial H_{eddy}^2}{\partial t} + \sigma \mu_0 \frac{\partial M}{\partial t}, \quad (15)
$$

$$
\nabla H_{eddy}^1 \cdot n = 0, \quad (16)
$$

and $H_{eddy}^2$ satisfies the Laplace equation

$$
\nabla^2 H_{eddy}^2 = 0 \quad (17)
$$

with the boundary conditions

$$
H_{eddy}^2 = 0, \quad r \rightarrow \infty. \quad (18)
$$

From the equations above we see that $H_{eddy}^2$ is the magnetic field of a magnetic dipole sheet. The solution can be written down in analogy to the potential solution of a magnetic dipole sheet

$$
H_{eddy}^2(r) = \int_{\partial \Omega} \mathbf{H}_{eddy}^2(r') \cdot \mathbf{n} \cdot \nabla \left( \frac{1}{r - r'} \right) \, da. \quad (20)
$$

III. NUMERICAL ALGORITHM

After setting up the conditional equations, we have to solve the system of differential and algebraic equations (DAEs) presented in Sec. II. This initial value problem is
solved using an implicit differential-algebraic (IDA) solver. The IDA solver uses the following form to solve this initial value problem:

\[ F(t, y, y') = 0, \quad (21) \]

where \( y \) and \( F \) are vectors in \( \mathbb{R}^N \) (in our case \( N=9 \)) of the form

\[ y_j = y_j \begin{bmatrix} m \\ \mathbf{H}_{\text{eddy}}^1 \\ \mathbf{H}_{\text{eddy}}^2 \end{bmatrix} \epsilon \mathbb{R}^3, \quad (22) \]

where \( t \) is the time and \( j \) is the node index with values from 1 to \( J \). The initial conditions \( y(t_0) = y_0, y'(t_0) = y'_0 \) are given.

Equation (21) is integrated using a backward differentiation formula (BDF) method implemented in a variable-order, variable-step form. The application of the BDF on the DAE system results in a nonlinear algebraic system that is solved with a Newton iteration. This leads to a linear system for each Newton correction that is solved with the scaled preconditioned generalized minimum residual method. Details of this method in combination with BDF can be found in Ref. 8.

IV. RESULTS

To test our model, we choose a large and instantaneously applied external field to get a fast switching of the magnetization in a reasonable time.

A Permalloy nanocube with an edge length of 20 nm was discretized in 1781 tetrahedrons with 468 surface elements. The LLG equation is solved for the nanocube that is initially magnetized uniformly in the \( z \) direction with an applied field of \( \mathbf{B}_{\text{ext}} = \mu_0 \mathbf{H}_{\text{ext}} = -10 \mu_0 \) T. Simulations were done with and without eddy currents for damping parameters of \( \alpha = 0.1 \) and \( \alpha = 0.5 \). The simulations show that the eddy currents result in an effective increase of the damping parameter as shown in Fig. 1. For the permalloy cube and \( \alpha = 0.1 \), the eddy current damping was found to be 0.02. However, this is an average value, as time dependent analysis of local magnetization vectors give the eddy current damping as a function of space. In addition the results show that with increasing damping parameter the contribution of eddy currents becomes less important, as shown in Fig. 2. The results obtained with this new model agree well with previously published results of eddy current effects in permalloy cubes.

V. CONCLUSIONS

A three-dimensional dynamic micromagnetic model which includes eddy currents was developed and applied to the simulation of a permalloy nanocube. The effect of eddy currents on the magnetization rate of change was investigated. It was noted that eddy currents result in an effective increase of the damping parameter \( \alpha \), and therefore in a faster switching of the system. In addition it was also shown that by increasing \( \alpha \) the effect of eddy currents becomes less important, as shown in Fig. 2.

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