Stiffness analysis for the micromagnetic standard problem No. 4

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In this article solutions to micromagnetic standard problem No. 4, a 500-nm×125-nm-wide NiFe film, are presented. A three-dimensional-finite element simulation based on the solution of the Gilbert equation has been used. The simulations show that two different reversal mechanisms occur for the two different applied fields. For a field at 170° counterclockwise from the saturation direction there is a nonuniform rotation of magnetization towards the direction of the applied field, with the magnetization at the ends rotating faster than the magnetization in the center. For a field at 190° counterclockwise from the saturation direction the magnetization at the ends and in the center rotate in opposite directions leading to the formation of a 360° wall after 0.22 ns associated with a peak in the exchange energy. Moreover, the time for the magnetization component parallel to the long axis to cross the zero is 0.136 and 0.135 ns for field 1 and field 2, respectively. The stiffness of the problem has been investigated solving the system of ordinary differential equations with a nonstiff method (Adams) and a stiff one (backward differentiation formula, BDF). For the measure of stiffness the ratio of the total number of time steps (nst) taken by the two solvers, that is nst(Adams)/nst(BDF), has been used. This ratio is 0.784 for field 1 and 0.593 for field 2, which means that the nonstiff method (Adams) uses larger time steps than the stiff method (BDF) and consequently the systems are not stiff. The average time step for the Adams method was 0.2 ps for both fields.
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I. INTRODUCTION

The magnetic material defined by micromagnetic (μMAG) standard problem No. 4 is a rectangle NiFe film, with thickness $t=3$ nm, width $d=500$ nm, and length $L=125$ nm. The initial state is an equilibrium *s* state. The *s* state is obtained after applying and slowly reducing a saturating field along the [1,1,1] direction to zero. Standard problem No. 4 is focused on the dynamic aspects of micromagnetic computations.[1] The problem has been studied using a three-dimensional-finite element simulation based on the solution of the Gilbert equation. The problem runs for two different applied fields, one at 170° (field 1) and the other at 190° (field 2) counterclockwise from the positive *x* axis. Figure 1 shows the magnetization distribution of the *s* state, the coordinate system and also the field directions.

The required output for the comparison is twofold: the *(x,y,z)* components of the spatially averaged magnetization of the sample as a function of time from $t=0$ until the sample reaches equilibrium in the new field, and also an image of the magnetization at the time when the *x* component of the spatially averaged magnetization first crosses zero. The magnetization images will be used to check for any differences in the reversal mechanisms if the time data between solutions are different.

II. MODEL AND SIMULATION METHOD

In micromagnetics the magnetic polarization is assumed to be a continuous function of space. The time evolution of the magnetization follows the Gilbert equation of motion

$$\frac{d\mathbf{J}}{dt} = -|\gamma_0|\mathbf{J}\times H_{\text{eff}} + \frac{\alpha}{J_s}\mathbf{J}\times\frac{\partial\mathbf{J}}{\partial t}, \tag{1}$$

which describes the physical path of the magnetic polarization **J** towards equilibrium. The effective field $\mathbf{H}_{\text{eff}}$ is the negative functional derivative of the total magnetic Gibb's free energy, which can be expressed as the sum of the exchange energy, the magneto-crystalline anisotropy energy, the magnetostatic energy, and the Zeeman energy.[2] The term γ_0 is the gyromagnetic ratio of the free electron spin and α is the damping constant. To solve the Gilbert equation numerically the magnetic particle is divided into finite elements. A hybrid finite element boundary element method[3] is used to calculate the scalar potential *u* on every node point of the

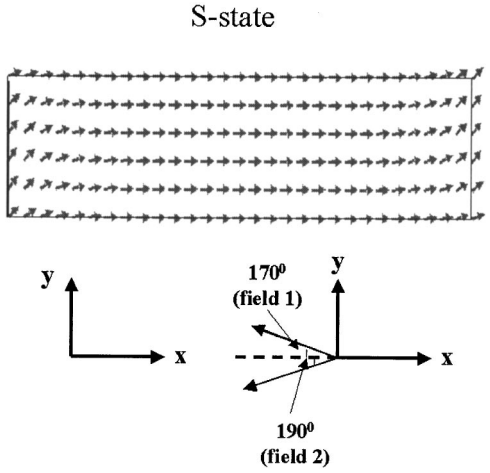

FIG. 1. Plot of the magnetization arrows for the *s* state, the coordinate system used and the direction of the two applied fields.

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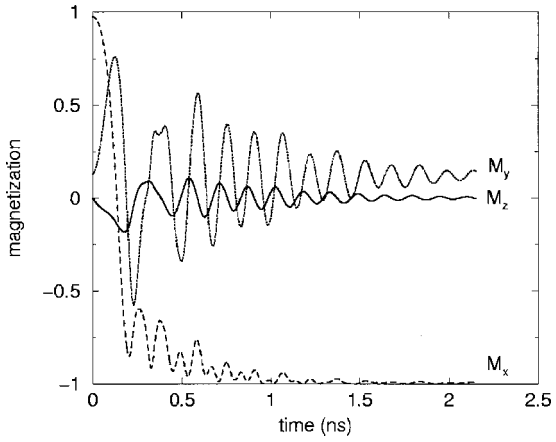

FIG. 2. Magnetization for the three components for field 1 during the simulation for ~2.2 ns (Adams method).

finite element mesh. The demagnetizing field, which contributes to the effective field, is the negative derivative of the scalar potential u. The effective field $H^i{}_{\text{eff}}$ at the node point i of an irregular finite element mesh can be approximated using the box scheme

$$\mathbf{H}_{\text{eff},i} = -\left(\frac{\delta E_t}{\delta \mathbf{J}}\right)_i = -\frac{1}{V_i}\frac{\partial E_t}{\partial \mathbf{J}_i}, \quad \text{for } V_i \to 0, \tag{2}$$

where V_i is the volume of the surrounding node i, such that

$$\sum_i V_i = V, \quad \text{and } V_i \cap V_j = 0 \quad \text{for } i \neq j. \tag{3}$$

The discretization of the Gilbert equation leads to an ordinary differential equation for every node for each component. In the case of a nonstiff problem it is advisable to use an appropriate method, such as Adams,[4] whereas in stiff problems a backward differentiation formula (BDF) method could be an option for the time integration. BDF method is implicit, so at each time step a nonlinear algebraic system must be solved. For the solution of the nonlinear system a method, such as Newton, has to be used which leads usually to a very large system of linear equations. In this article the latter is solved using the scaled preconditioned incomplete

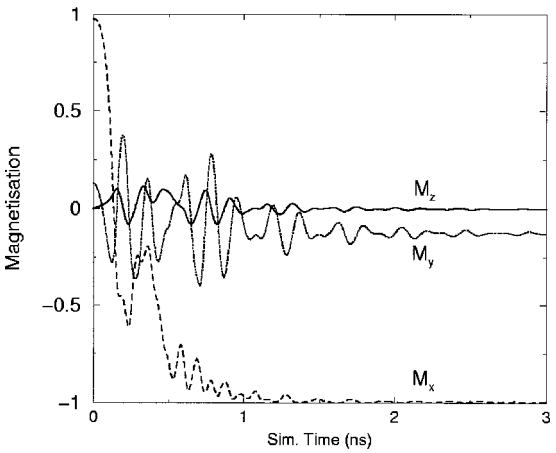

FIG. 3. Magnetization for the three components for field 2 during the simulation for ~3 ns (Adams method).

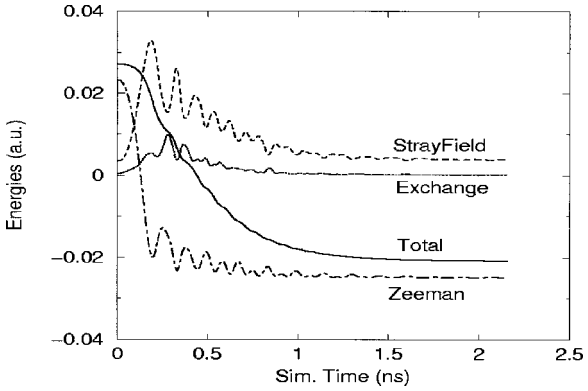

FIG. 4. The stray field, Zeeman, exchange and total energy for field 1 during the simulation for up to ~2.2 ns (Adams method).

generalized minimum residual method (SPIGMR),[5] based on the generalized minimum residual method proposed by Saad.[6] SPIGMR belongs to the family of Krylov subspace methods, which are iterative methods for solving systems of linear equations. SPIGMR has been explored in micromagnetics by Tsiantos, Miles, and Middleton,[7,8] and also used by Yang and Fredkin.[9]

The stiffness of the problem has been investigated solving the system of ordinary differential equations with two different solvers. A nonstiff method (Adams) and a stiff one (backward differentiation formula, BDF) have been used to measure the stiffness of the problem. For the latter the ratio of the total number of time steps (nst) taken by the two

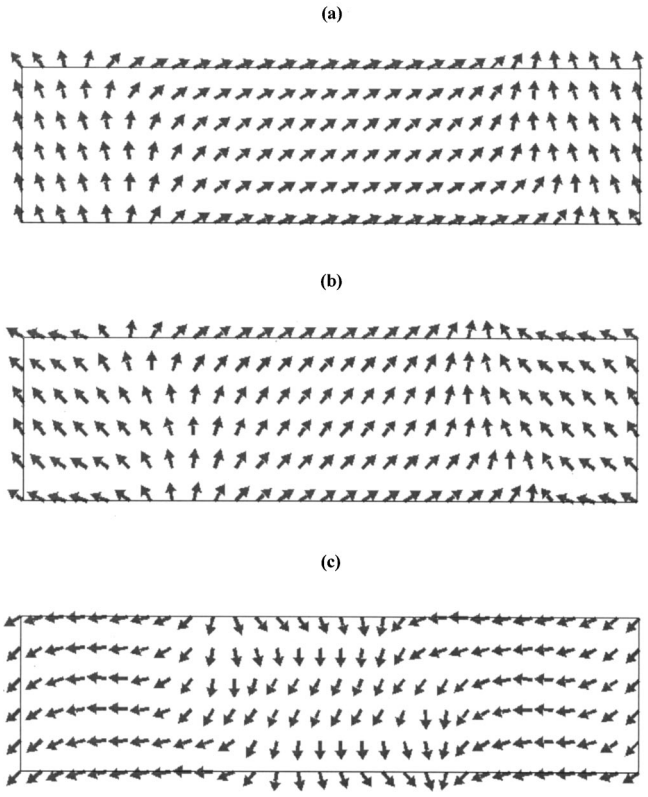

FIG. 5. Plot of the magnetization arrows for field 1: (a) before M_x crosses the x axis (0.107 ns), (b) when M_x crosses the x axis (0.136 ns), and (c) after M_x crosses the x axis (0.241 ns).

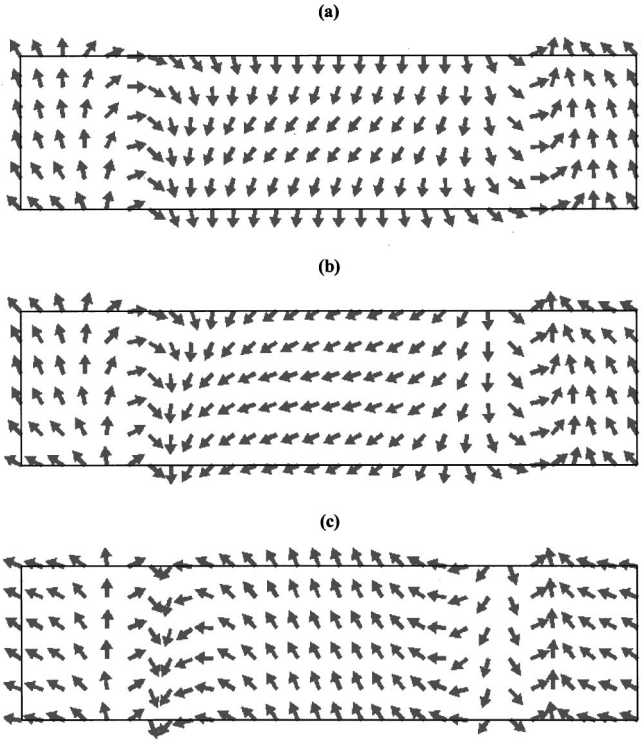

FIG. 6. Plot of the magnetization arrows for field 2: (a) when M_x crosses the x axis (0.135 ns), (b) 0.151 ns, and (c) 0.198 ns from the beginning of the simulation.

solvers, that is nst(Adams)/nst(BDF), has been used. Note that the simulation time has to be the same in order to have a fair comparison. The above mentioned method has been proposed to approximate numerically the stiffness of a system of ordinary differential equations (ODEs) in micromagnetics by Tsiantos and Miles.[10] Another factor that has to be considered is the cost of each method per time step. This cost per time step is important in cases that the ratio nst(Adams)/nst(BDF) is larger than 1. If the ratio of time steps is smaller than 1 then the case is nonstiff. For the Adams the main cost per time step is the function evaluation. However, for the case of BDF there is some extra cost for the linear algebra involved. This extra cost is due to linear and nonlinear iterations. In general, the preconditioned case gives faster results in terms of the nonlinear and linear iterations. However, the preconditioned method roughly doubles the average cost per nonlinear iteration because it computes and processes the preconditioner.[5]

III. RESULTS

The simulations show that two different reversal mechanisms occur for the different fields. For field 1 there is a nonuniform rotation of magnetization towards the direction of the applied field, with the magnetization at the ends rotating faster than the magnetization in the center. For field 2 the magnetization at the ends and in the center rotates in opposite directions leading to the formation of a 360° wall after 0.22 ns associated with a peak in the exchange energy. Moreover, the time for the x component of the magnetization to cross the x axis is 0.136 and 0.135 ns for field 1 and field 2, respectively. Figures 2 and 3 give the evolution of the magnetic components for the different fields. Furthermore, after reversal the magnetization oscillates with decreasing amplitude. Figure 4 shows how the micromagnetic energy contributions evolve in time (field 1). After the x component of the magnetization crosses zero, an energy transfer occurs between the stray field energy and the Zeeman energy. Figures 5 and 6 show the magnetization distribution when m_x crosses the x axis (required by the definition of the problem), as well as at two other time moments for fields 1 and 2, respectively.

With regard to the stiffness of the problem the ratio of the total number of time steps taken by the two solvers, that is nst(Adams)/nst(BDF), is 0.784 for field 1 and 0.614 for field 2, which means that the nonstiff method (Adams) uses larger time steps than the stiff method (BDF) and consequently the systems are not stiff. The total number of the time steps taken by each method for field 1 is 11 473 (Adams) and 14 628 (BDF). For field 2 we have that nst(Adams) $=11\,342$ and nst(BDF)$=18\,479$. The simulation time considered for field 1 is 2.15 ns and for field 2 is 2.23 ns. The average time step for the Adams method was 0.2 ps for both fields.

The μMAG problem No. 4 can be misleading with regards to its stiffness because the Adams method takes too many time steps. Thus, it can be thought of as a stiff case if it will not be compared to a stiff method. The possible explanation for the large number of time steps is the low value of the damping constant used, $\alpha=0.02$. The low value of α causes the magnetization to move around the effective field so the time integrator needs very small time steps to follow the path of the magnetization.

For the ODE solver we used mixed error criterion with absolute and relative tolerance equal to $10e$-4. Moreover, after 2.42 ns for field 1 and 2.23 ns for field 2 the amplitude of the oscillations of the magnetization obtains the requested numerical accuracy.

ACKNOWLEDGEMENT

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